ECE 582 Homework 1

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1. We know $P_R = 1$ dBm, $G_l = 1$, $f_c = 5E9$ Hz. Find the transmitted power P_T at $d = \{10, 100\}$ m using the free space path loss model. The FSPL model is

$$P_R \text{ dBm} = P_T \text{ dBm } + 10\log_{10} G_l + 20\log_{10}\lambda - 20\log_{10} 4\pi - 20\log_{10} d.$$

$$\begin{aligned} d = 10\text{m}: \qquad 1 &= P_T \text{ dBm } + 10\log_{10} 1 + 20\log_{10}\frac{c}{5E9} - 20\log_{10} 4\pi - 20\log_{10} 10. \\ 1 &= P_T \text{ dBm } + 0 + (-24.44) - 21.98 - 20 \\ 1 &= P_T \text{ dBm } - 66.42 \\ P_T \text{ dBm} &= \boxed{65.42} \end{aligned}$$

$$\begin{aligned} d = 1\text{m}: \qquad 1 &= P_T \text{ dBm } + 10\log_{10} 1 + 20\log_{10}\frac{c}{5E9} - 20\log_{1} 4\pi - 20\log_{10} 10. \\ 1 &= P_T \text{ dBm } + 0 + (-24.44) - 21.98 - 40 \\ 1 &= P_T \text{ dBm } - 86.42 \\ P_T \text{ dBm} &= \boxed{85.42} \end{aligned}$$

2. This question involves finding an approximation for $\Delta\phi$ for large values of d relative to $(h_t + h_r)$ so that

$$\Delta\phi = \frac{2\pi(x + x' - l)}{\lambda} \Rightarrow \frac{4\pi h_t h_r}{d\lambda}$$

First we will define the function $f(d)$ as equation 2.13, so

$$f(d) = (x + x' - l) = \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}$$

Next we notice that for the approximation to work, we need $f(d) \approx \frac{2h_t h_r}{d}$. As d goes large relative to $(h_t + h_r)$, we have

$$\lim_{d\to\infty} f(d) = 0$$

If we define the function $g(d)$ as follows, we see that

$$g(d) = \frac{2h_t h_r}{d} = \frac{h_t h_r}{d} + \frac{h_t h_r}{d}, \qquad \lim_{d\to\infty} g(d) = 0$$

And so at high d values, $g(d) \approx f(d)$, and so

$$\Delta\phi = \frac{2\pi f(d)}{\lambda} \approx \frac{2\pi g(d)}{\lambda} \approx \frac{4\pi h_t h_r}{d\lambda}$$

3. Given a channel response of $h(t) = \alpha_1\delta(t) + \alpha_2\delta(t - 0.022\mu s)$, a carrier frequency $f_c = 900$ MHz, $h_t = 8 = h_r$ and a reflection coefficient of -1, find the distance between the receivers and α_1, α_2.

If we assume a propagation velocity of c, we can determine the delay distance from the delayed impulse time of $0.022\mu s$. If $x + x'$ is the reflected path and l the direct path, we have

$$\begin{aligned} \frac{x + x' - l}{c} &= 0.022\mu s \\ x + x' - l &= 6.6\text{m} \end{aligned}$$

Next we can solve $f(d) = 6.6$ for d.

$$\begin{aligned} f(d) = 6.6 &= \sqrt{(8+8)^2 + d^2} - \sqrt{d^2} \\ 6.6 &= \sqrt{(16)^2 + d^2} - \pm d \\ (6.6 + d)^2 &= 16^2 + d^2 \\ 13.2d &= 16^2 - 6.6d \\ d &= \boxed{16.09 \text{ m}} \end{aligned}$$

To calculate the channel response coefficients, we use the two-ray path loss model. Also, assume a unity transmitter power of 1W, $\gamma = 4$, and far-field reference $d_0 = 10$. With these parameters, K (dB) is computed to be -51.53 dB from equation (2.41). We first need to calculate the received power for both the reflected and direct path. Here, I am using equation (2.40). So with this we have

$$\begin{aligned} d = 16.09\text{m}: \quad P_r &= 10\log_{10}(1000) \text{ dBm } - 51.53 \text{ dB} - 10 \cdot 4\log_{10}(d = 16.09/10) \\ &= -29.79 \text{ dBm} \end{aligned}$$

$$\begin{aligned} d = 16.09 + 6.6\text{m}: \quad P_r &= 10\log_{10}(1000) \text{ dBm } - 31.53 \text{ dB} - 10 \cdot 4\log_{10}(d = 22.69/10) \\ &= -35.76 \text{ dBm} \end{aligned}$$

Converting these power measurements to non-decibel and then taking the ratio relative to transmitted power gives us the final attenuation constants. Since $R = -1$, α_2 will be negative.

$$\begin{aligned} d = 16.09\text{m}: \quad \alpha_1 &= \frac{P_R}{P_T} \\ \alpha_1 &= \frac{10^{-2.979}}{1000} \\ \alpha_1 &= \boxed{\text{1.05E-6}} \end{aligned}$$

$$\begin{aligned} d = 22.69\text{m}: \quad \alpha_2 &= -\frac{P_R}{P_T} \\ \alpha_2 &= -\frac{10^{-3.576}}{1000} \\ \alpha_2 &= \boxed{\text{-2.65E-7}} \end{aligned}$$

4. Using the Hata model, assuming $f_c = 900$ MHz, $h_t = 20$ m, $h_r = 5$ m and $d = 100$ m, find the path loss for a large urban city, small urban city, a suburb, and a rural area.

   (a) For the large urban city, the model is

   $$P_L \text{ dB } = 69.55 + 26.16\log_{10}(f_c) - 13.82\log_{10}(h_t) - a(h_r) + (44.9 - 6.55\log_{10}(h_t))\log_{10}(d)$$

   where for large cities are high frequencies, $a(h_r)$ equals

   $$a(h_r) = 3.2(\log_{10}(11.75h_r))^2 - 4.97 \text{ dB}$$

   Substituting:

   $$\begin{aligned} P_L &= 69.55 + 26.16\log_{10}(900E6) - 13.82\log_{10}(20) - [3.2(\log_{10}(11.75 \cdot 5)^2 - 4.97] + \\ &\quad + \big(44.9 - 6.55\log_{10}(20)\big)\log_{10}(100) \\ &= 69.55 + 234.243 - 17.98 - [10.01 - 4.97] + 72.756 \\ P_L &= \boxed{353.529 \text{ dB}} \end{aligned}$$

   (b) For the small urban city, the only change is $a(h_r)$:

   $$a(h_r) = (1.1\log_{10}(f_c) - 0.7)h_r - (1.56\log_{10}(f_c) - 0.8) \text{ dB}$$

   Substituting:

   $$\begin{aligned} P_L &= 69.55 + 234.243 - 17.98 - \\ &\quad - [(1.1\log_{10}(900E6) - 0.7) \cdot 5 - [1.56\log_{10}(900E6) - 0.8] + 72.756 \\ &= 69.55 + 234.243 - 17.98 - (45.748 - 13.168) + 72.756 \\ P_L &= \boxed{325.989 \text{ dB}} \end{aligned}$$

   (c) The suburb model:

   $$\begin{aligned} P_L &= P_{L,\text{urban}}(d) - 2[\log_{10}(f_c/28)]^2 - 5.4 \\ &= 353.529 - 2[\log_{10}(900E6/28)]^2 - 5.4 \\ P_L &= \boxed{235.416 \text{ dB}} \end{aligned}$$

   (d) Lastly for the rural model, with $K = 35.94$:

   $$\begin{aligned} P_L &= P_{L,\text{urban}}(d) - 4.78[\log_{10}(f_c)]^2 + 18.33\log_{10}(f_c) - K \\ &= 353.529 - 4.78[\log_{10}(900E6)]^2 + 18.33\log_{10}(900E6) - 35.94 \\ P_L &= \boxed{97.467 \text{ dB}} \end{aligned}$$

   As the scarcity of buildings and obstructions increases, the obstacles to clear reception decrease. Hence the path loss decreases as we move form urban environments to rural locations.

5. Find the maximum cell size that will allow 90% access at the prescribed level, given free space path loss (therefore $\gamma = 2$) coupled with log-normal fading with $\sigma = 6$ dB. Cell system with

$f_c = 900$ MHz, an SNR of 15 dB, a $P_T = 1$ W with 3 dB gain. Noise in the bandwidth is -40 dBm.

Given the SNR and the noise level, the received power is $-40 + 15 = -25$ dBm. The transmitter has a 1W transmit power, or 30 dBm. With the antenna gain of 3 dB, the EIRP of the antenna is $P_T = 33$ dBm. Based on the presumption that we're working outside at 900MHz, we'll use the far field factor be $d_0 = 10$. With these values, K is calculated to be $K = -51.61$ dB.

In this problem, we're looking for an outage probability of 10%. Since this is log-normal shadowing, we use equation 2.52 from the text. Further, by use of Z-Tables, we note that

$$1 - Q(-1.2815) = 0.1$$

So now we have, in generality

$$0.1 = 1 - Q\left[\frac{-25 \text{ dBm } - (33 \text{ dBm } - 51.61 \text{ dB} - 10 \cdot 2\log_{10}(d/10))}{6}\right]$$

The argument of the Q function needs to equal -1.2815:

$$\begin{aligned}
\frac{-25 \text{ dBm } - (33 \text{ dBm } - 51.61 \text{ dB} - 10 \cdot 2\log_{10}(d/10))}{6} &= -1.2815 \\
20\log_{10}(d/10) &= 14.079 \\
\log_{10}(d/10) &= .704 \\
d/10 &= 5.06 \\
d &= \boxed{50.58 \text{ m}}
\end{aligned}$$

6. A cellular system with log-normal distributed shadowing, with mean μ dBm and standard deviation σ_Ψ dBm. The received signal power must be above 10 dBm for acceptable performance.

   (a) $\mu_\Psi = 15$ dBm and $\sigma_\Psi = 8$ dBm .

   $$Q\left(\frac{10-15}{8}\right) = Q(-0.625) = \boxed{0.734}$$

   This is the probability that the signal will be above 10 dBm, so for the outage probability it is simply

   $$P = 1 - 0.734 = \boxed{0.266}$$

   (b) $\sigma_\Psi = 4$ dB. Find a μ value $\ni$ the outage probability is less than 1%. First, note that 1-Q(-2.33) = 0.01. We then have

   $$\begin{aligned}
   -2.33 &= \left(\frac{10-\mu}{4}\right) \\
   -9.32 &= 10 - \mu \\
   \mu &= \boxed{19.32 \text{ dB}}
   \end{aligned}$$

(c) Same deal as (b), but $\mu_\Psi = 12$ dBm.

$$\begin{aligned} -2.33 &= \left(\frac{10-\mu}{12}\right) \\ -27.96 &= 10-\mu \\ \mu &= \boxed{37.96 \text{ dB}} \end{aligned}$$

(d) This would reduce outage probability simply because the odds of all of the towers going out at the same time is very very low. Adding the additional towers simply adds redundancy to the network, so if one tower went offline there would be others there to route the traffic.